Title: Highs and Lows

Brief Overview:

This unit teaches students to use the TI-83 to find the relative extrema of a function. The students will be able to determine extrema by using a graph, a table, and the values of the derivative.

Links to NCTM 2000 Standards:

- **Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation**
  These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

  Students will locate the extrema of a function, and they will read tables and analyze the behavior of the derivative at extrema. They also will use correct mathematical language to describe results. In addition, students will recognize the relationship between the graph of the function and the graph of the derivative. Last of all, they will construct tables and graphs using the TI-83 to discover relationships between the function and the derivative.

- **Patterns, Functions, and Algebra**
  Students will discover the pattern of the derivative around an extrema.

- **Data Analysis, Statistics, and Probability**
  Students will analyze the data in tables to make conclusions about the relationship of a function and its derivative.

Links to Maryland High School Mathematics Core Learning Units:

  Functions and Algebra
- **1.1.1**
  Students will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically, and geometrically.

- **1.1.2**
  Students will represent patterns and functional relationships in a table, as a graph, and/or by mathematical expression.

- **1.1.4**
  Students will describe the graph of a non-linear function in terms of the basic concepts of maxima and minima, roots, limits, rates of change, and continuity.

- **1.2.4**
  Students will describe how the graphical model of a non-linear function represents a given problem and will estimate the solution.

  Data Analysis and Probability
- **3.1.1**
  Students will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results.

- **3.2.1**
  Students will make informed decisions and predictions based upon the results of simulations and data from research.

Links to National Science Education Standards:

- **Life Science**
  Students will use knowledge of the derivative to solve a problem involving the rate of absorption of a drug into the blood stream.

Grade/Level:

This learning unit is appropriate for students in Algebra II, Pre-Calculus, and Calculus.

Duration/Length:

Two days will be necessary for the lesson, and one day will be needed for the closure and assessment.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- General knowledge of functions and appropriate terminology
- General knowledge of using the TI-83

Student Outcomes:

Students will:

- use the TI-83 to find relative extrema by using trace, tables, and the nDeriv function.

Materials/Resources/Printed Materials:

- TI-83
- TI-83 overhead projector
- Activity Sheets

- Practice Quiz
- Assessment
- Student Answer Sheet

Development/Procedures:

The teacher will introduce the lesson with a motivator about an amusement park. These self-directed activities are designed to be used in a calculator AB class with minimal teacher direction. The teacher's role will be to summarize and review skills and concepts. An Algebra II class, working with maxima and minima should limit activities to Section 1.

Assessment:

A practice quiz is given using roller coasters. Finally, an assessment is given which covers all of the acquired skills and uses them in a practical Biology application. Questions are given in Selected Response, Brief Constructed Response, and Extended Constructed Response form.

Extension/Follow Up:

After completing this unit, students will be ready to complete applied maxima and minima problems.

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TEACHER PREPARATION GUIDE

Prior to beginning any calculator lesson, all calculators should be reset to default values to erase any changes made by the games students play. This can be accomplished using the following steps:

1. Press **2nd MEM**
2. Select **5: Reset**
3. Select **2:Defaults**
4. Select **2:Reset**

This procedure will not erase the calculator memory, but will reset the calculator to the default parameters originally set at the factory.

When working with graphs of functions it would be wise to give students experience at changing the window of the graph to be able to see all-important components. In problem number 6 the window should be set as follows:

Xmin $= -2\pi$
Xmax $= 2\pi$
Xscl $= \dfrac{\pi}{12}$
Ymin $= \mathbf{-10}$
Ymax $= 10$

The calculator will often fail to give exact integer answers. Realize that .999999999 should really be 1 and values such as .0000345E-8 is actually 0.

A Day at FUNction Land

 The people of ancient Mathepotamia, where everyone counts, spent much of their vacation time at an amusement park called FUNction Land. Like most amusement parks today, the favorite ride at the park was the roller coaster. The TI-83 Function Coaster was set up to follow the equation of a function which was changed each time you rode on it. At the end of each ride, you had to take a simple quiz which asked for all the relative extrema- maximums and minimums. At the end of the first ride, you were given the following quiz:

"You are now at the end of your ride. In order to disembark, you must determine any relative maximums or minimums for the function $f(x) = -x^2 - 6x - 7$. If you are having difficulty completing this task, please refer to the manual for finding ride extrema that was given to you upon entering the park. If you determine the correct answer, you will be given a token for a free ride. Remember that each time you ride the coaster the function and therefore the path is changed. With each ride, the difficulty of the function increases.

Have fun and enjoy the rides! Come back and visit us as often as you like. Be sure to also visit our other attractions. Take a ride on the **Relation Ship**. Slide down the **Decreasing Function**. Ride the **Parabola of Revolution**. "
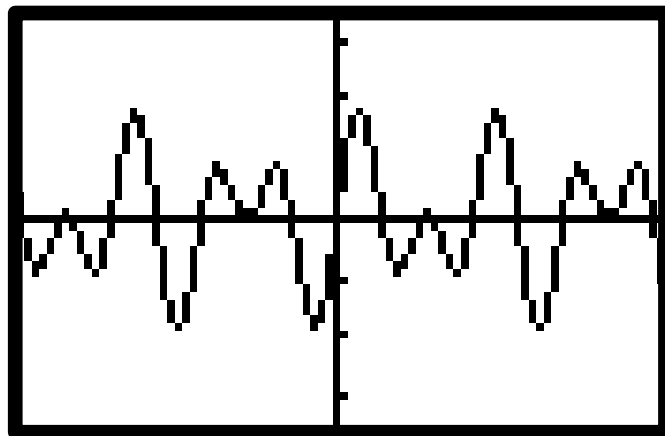
THE TI-83 FUNCTION COASTER

HIGHS and LOWS

Your Guide to Finding Relative MAXIMUMS and MINIMUMS
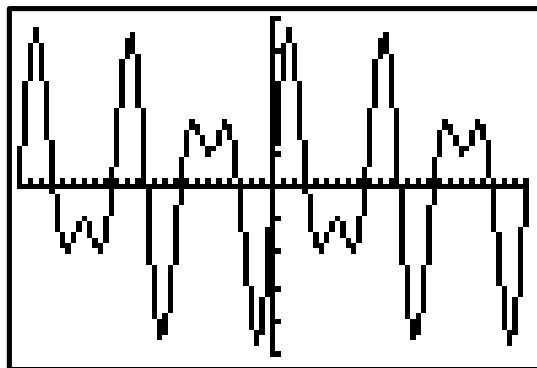
A
Manual for Using the TI-83
Graphing Calculator
to
Find Function Extrema
.

## ACTIVITY 1: FINDING A FUNCTION'S MAXIMUM OR MINIMUM

This lesson will use the TI-83 calculator to determine the maximum or minimum of a function using a table, a graph, or nDeriv. It will also relate the use of the derivative to determine the maximum and minimum of a function.

$$f(x) = -x^2 - 6x - 7$$

A. Using the calculator max-min functions.

I. Using the trace function

1. Enter the function using the **Y=** button.
2. Press the **GRAPH** to view the function graph on Z6.

3. Observe the graph.
   Does it have a minimum or maximum? _____
   At approximately what value of **x** does that point occur? _____
   Estimate the value of the function at this **x**. _____

4. Use the **TRACE** button to approximate the value of **x**. _____
   Use the **TRACE** button to approximate the value of **y**. _____

5. Algebraically (rewrite the equation in the form $y = a(x - b)^2 + c$)
   determine the vertex.
   x = _____   **y** = _____
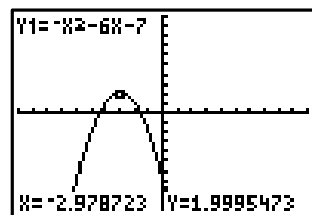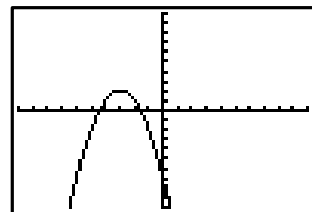
II. Using the table.

1. Press the **MODE** button and set the mode to split screen **G-T**, graph and table. This will display both a function table and draw the function.

2. Press the **TRACE** and use the left/right keys to get close to the maximum. Read the values of **x** and **y** from the table.
   x = _____   **y** = _____

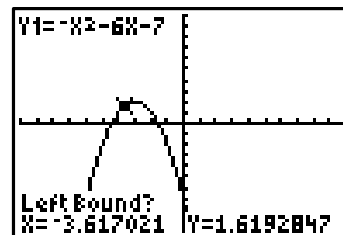### III. Using the CALC function.

1. Reset the mode to full screen.
2. Graph the function.
3. Press **CALC** key (2nd **TRACE**)
4. Select option **4:maximum**.
5. Use the left arrow to select **LEFT BOUND?**
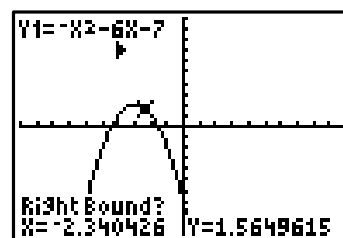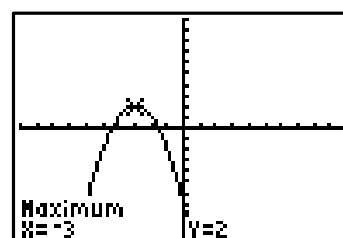   to the left of the point. Press **ENTER**.

6. Use right arrow to **RIGHT BOUND?**
   to the right of the point. Press **ENTER**.

7. Press **ENTER** again to record a guess.
   (No number is necessary, just press left arrow once to
   enter a guess.)

8. Read the maximum value of the function and where it
   occurs. **x** = _____   **y** = _____

B. Finding the maximum or minimum on an open interval (relative max or mins)

A relative minimum or relative maximum is the smallest or largest **y** value on a given interval of **x**. The function $f(x) = x^4 - 2x^2 - 6$ has 2 relative minimums and one relative maximum.
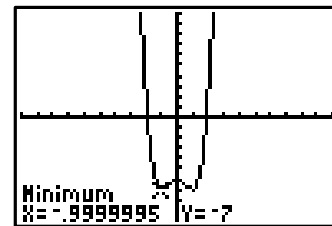
   1. Enter the function using **Y =**.
   2. Graph the function on **Zoom 6**.
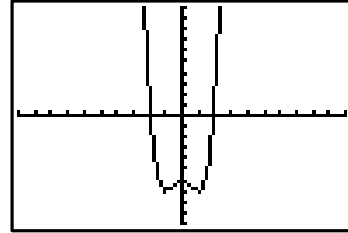
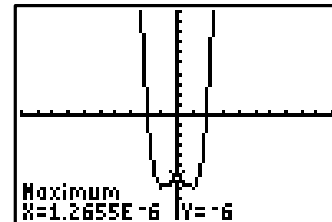   3. Find the relative minimum on the interval $(-\infty, 0)$.
       Use **CALC 3**.
       (Remember about the left and right bounds)
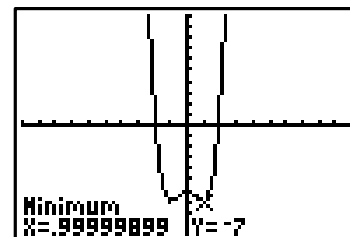   **x** = _____   **y** = _____

   4. Repeat the procedure for finding the relative maximum on
         (-1, 1). Use the **CALC 4** function.

       x = _____   **y** = _____

   5. Find the relative minimum on the interval $(0, \infty)$. Use
       CALC 3.

       x = _____   **y** = _____

   6. See extension for problem with maximum or minimum
       on a <u>closed</u> interval.

ACTIVITY 2: MAXIMUMS, MINIMUMS, AND DERIVATIVES.

I. Using the dy/dx function.

1. Enter the $f(x) = -x^2 - 6x - 7$ into **Y=**.
2. Graph the function.
3. Press **2nd CALC**
4. Select **6**
5. Use the left arrow to position the cursor on the
   X-axis to the left of the maximum point.
6. Press **ENTER**
7. Read dy/dx
8. Enter the data in the chart below.
9. Press **2nd CALC**, move the cursor closer to the
   Maximum and press **ENTER**.
10. Repeat until you reach the maximum that you
    Determined earlier.
11 Repeat the steps for the right side of the maximum.

```
CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
```

dy/dx=2.9361702
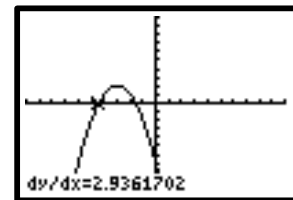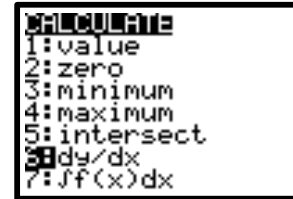
DATA CHART

x	dy/dx
-3	

12. What happens to the value of the derivative, **dy/dx**, as x approaches the maximum point?
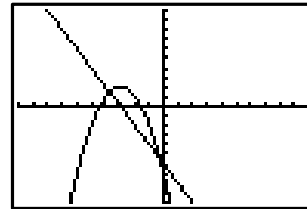
II. Using the nDeriv function

The TI-83 calculator enables you to calculate the numerical value of the derivative of a function using **nDeriv (MATH 8).** The function is used as **nDeriv**(function, argument, value). To calculate the derivative of $f(x) = x^2$ at **x=2**, you would enter $nDeriv(Y_1, x, 2)$. Using the graph and table functions you will compare the graph of the function and derivative and see the values of each in a table.

1. Enter **Y=.**
2. Select **Y$_2$**
3. Press **MATH 8**.
4. Press **VARS.**
5. Move cursor to **Y-VARS**.
6. Press **ENTER**.
7. Select **Y$_1$** and press **ENTER**
8. Enter **x,x) Enter**
9. Graph the functions.
10. Compare the graph of the function **(Y$_1$)** and the Graph of the derivative **(Y$_2$)**.

What is the value of the derivative at the maximum?

III. Using the table.

1. Go to table set up by pressing **2nd TBLSET**.
2. Set table parameters **TblStart = -5 ΔTbl = .5** .
3. Column **X** has the value of **x, Y$_1$=** function value, **Y$_2$=** derivative value.
4. Use the up and down arrows to scroll through the table.

What is the value of the derivative at the function maximum?

Extension: Find the maxima and minima on a closed interval. Find the extrema on [-1, 4] for $y = x^2 + 3$. Be sure to check the endpoints.

1. Minimum **x** = _____ **y** = _____
2. Maximum **x** = _____ **y** = _____

.

Activity 1.
AI.
3. The graph has a maximum (absolute).
   The maximum occurs at $x = -3$.
   The function value is 2.

4. Answers will vary with each student.
5. $x = -3$, $y = 2$

AII.
2. These answers will vary.

AIII.
8. $x = -3$, $y = 2$

B.
3. $x = -1$ $y = -7$ (Be aware that the calculator often has trouble with certain integers.)
4. $x = 0$, $y = -6$
5. $x = 0$, $y = -7$

Activity 2.

I.
The chart data will very depending upon the location of the cursor. Students should conclude that the derivative of the function approaches 0 as the function approaches a maximum or minimum value.

II.

A.
The derivative has a value of 0 at the maximum.

B.
The Y_2 column has a value of 0 when then function equals 2.

Extension

1. Minimum $x = 0$ $y = 3$
2. Maximum $x = 4$ $y = 19$

THE TI-83 FUNCTION COASTER QUIZ SET

Problem #1

The original problem was in Activity 1.

Problem #2

$$\mathbf{f(x)} - \mathbf{x}^4 - 2\mathbf{x}^2 - 6$$

Problem #3

$$\mathbf{f(x)} = \frac{\mathbf{x}^3}{3} - \frac{\mathbf{x}^2}{2} - 2\mathbf{x} + \frac{1}{3}$$

Problem #4

$$\mathbf{f(x)} = \mathbf{x}^4 + \mathbf{x}^3 - 3\mathbf{x}^2 + 1$$

Problem #5

$$\mathbf{f(x)} = 2|\mathbf{x} - 1| + 3$$

Problem #6

$$\mathbf{f(x)} = \mathbf{x} + \sin 2\mathbf{x}$$

Problem #7

$$\mathbf{f(x)} = \mathbf{x}^{\frac{2}{3}}(\frac{5}{2} - \mathbf{x})$$

ANSWER KEY FOR THE TI-83 COASTER QUIZ

Problem #2

The function $f(x) - x^4 - 2x^2 - 6$ has 2 local minimums $f(x) = -7$ at $x = -1$ and $x = 1$ and a local maximum $f(x) = -6$ at $x = 0$.

Problem #3

The function $f(x) = \dfrac{x^3}{3} - \dfrac{x^2}{2} - 2x + \dfrac{1}{3}$ has a local minimum $f(x) = -3$ at $x = 2$ and a local maximum $f(x) = -1$ at $x = \dfrac{3}{2}$.

Problem #4

The function $f(x) = x^4 + x^3 - 3x^2 + 1$ has 2 local minimums $f(x) = -4.248$ at $x = -1.656$ and $f(x) = -.045$ at $x = .906$ and a local maximum $f(x) == 4.248$ at $x = -1.656$.

Problem #5

The function $f(x) = 2|x-1| + 3$ has absolute minimum $f(x) = 3$ at $x = 1$.

Problem #6

There are 4 relative minimums $f(x) = -5.055, -1.913, 1.228, 4.390$ at
$x = 4.198, 1.047, 2.094, 5.236$ respectively.
The function $f(x) = x + \sin 2x$ has 4 local maximums,
$f(x) = -4.370, -1.228, 1.913,$ and 5.055 at $x = -5.236, -2.094, 1.048, 4.189$ respectively.

Problem #7

The function $f(x) = x^{\frac{2}{3}}(\dfrac{5}{2} - x)$ has a relative minimum $f(x) = 1.625$ at $x = 0$ and one relative maximum at $f(x) = \dfrac{3}{2}$ at $x = 1$.

Assessment for Highs and Lows
Teacher's Guide

Introduction

This assessment should follow the Highs and Lows learning unit. Each student will need a copy of the assessment, an answer page, a graphing calculator, and a pencil. The assessment consists of three parts:

Part A	Selected Response	5 questions at 1 point each
Part B	Brief Constructed Response	6 questions at 2 points each
Part C	Extended Constructed Response	1 question at 8 points

25 points

Assessment
Highs and Lows

Directions: Choose the best response for each question, and put your answer on the answer sheet. Do not write on this test.

A. Selected Response

1. Find the relative minimum of the function $2x^3 + 3x^2 - x + 1$.

   a) $x = 0$ b) $x = .145$ c) $y = .873$ d) $y = .924$

2. Determine the number of relative extrema the function $\dfrac{x^2 - 2x + 3}{x}$ has on the interval $(0, \infty)$.

   a) 0   b) 1   c) 2   d) 3

3. Approximate the value of the derivative at x = 2.
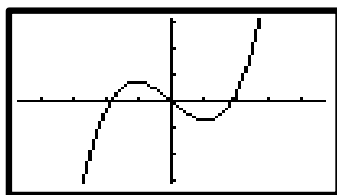
   a) $-.1$ b) 0   c) $.1$ d) $.9$

4. For $f(x) = (x - 3)^2$, determine what is happening to the value of the derivative on $(-\infty, 3)$.

   a) constant   b) decreasing   c) increasing   d) DNE

5. For $y = |x + 2|$, $f'(2) = $____.
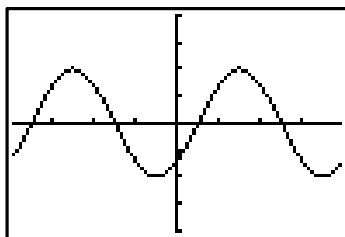
   a) -1   b) 0   c) 1   d) DNE

B. Brief Constructed Response

For questions 1-3, sketch the derivative of the given function on the axis provided on the answer sheet.
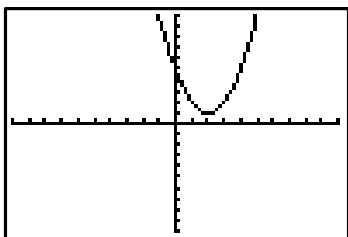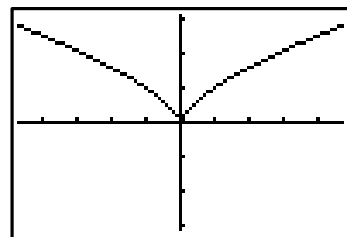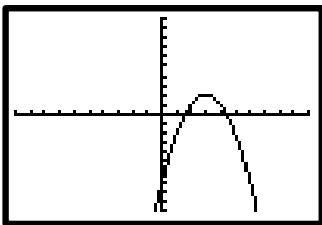
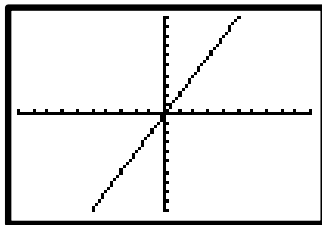1. $y = (x - 2)^2 + 1$ 2. $y = \sin(x - p/4)$ 3. $y = x^{\frac{2}{3}}$

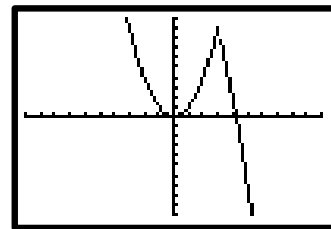For questions 4-6, find the value of the derivative of the given function at x = 3.

4.

5.

6.

C. Extended Constructed Response

A pharmaceutical company has developed a pain reliever where concentration C in the bloodstream t hours after ingestion is given by the function $C = \dfrac{t}{8 + t^3}$.

a) Graph the concentration in an appropriate viewing window. Consider the variables before constructing your graph. Label your axes and scale.
b) When is the concentration greatest?
c) At $t = 2.5$ hours, is the concentration increasing or decreasing?
d) Find the slope of the function at $t = .5$ hour and $t = 1$ hour. Use these answers to determine when the concentration is increasing the fastest.
e) Suppose the pharmaceutical company was making a drug to relieve heart attacks, severe migraines, or some other ailment that needs immediate relief. Explain what the slope of the concentration function should look like in the first hour.

Assessment

Highs and Lows

Name _____

Date _____ Period _____

A. 1. _____

    2. _____

    3. _____

    4. _____

    5. _____

B.

1.
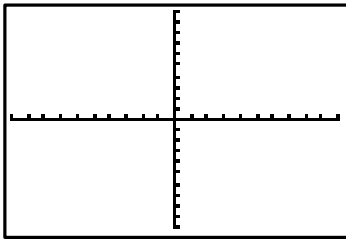
2.
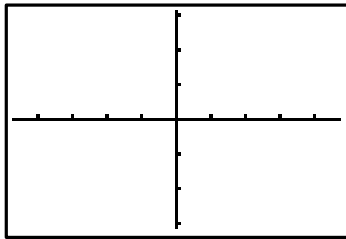
3.
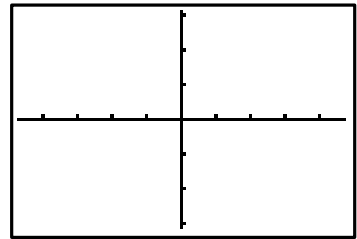
4. _____

5. _____

6. _____

C.

ANSWER SHEET

Assessment   **Name** _____

Highs and Lows   **Date** _____ **Period** _____

A.   1.   _____b_____

     2.   _____b_____

     3.   _____d_____

     4.   _____b_____

     5.   _____c_____

B.

1.   2.   3.
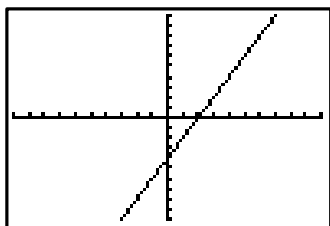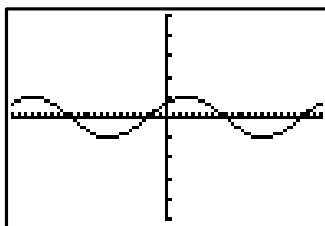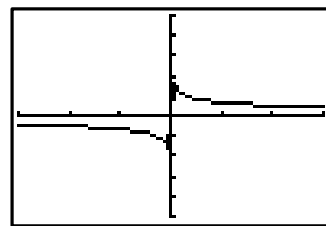
   4.   _____0_____   5.   _____2_____   6.   ___DNE_____

C.   concentration
   a.
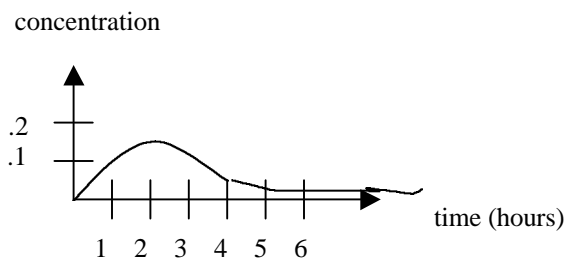
   b.   1.59 hr.
   c.   decreasing function
   d.   f'(.5)=0.117; increasing more rapidly at .5 hr. than 1 hr.
        f'(1)=0.074
   e.   slope should increase rapidly

SCORING RUBRIC FOR EXTENDED CONSTRUCTED RESPONSE

a. 1   Graph is drawn correctly. Labels are correct and the scale is appropriate.
    0   Graph is not correctly drawn.

b. 1   The answer is correct.
    0   The answer is not correct

c. 1   The answer is correct.
    0   The answer is not correct.

d. 3   Both slopes were determined correctly and the time was determined
               correctly.
    2   One correct slope was determined and the time was determined correctly.
    1   Either the slope or the time was incorrectly determined.
    0   Both answers were incorrect.

e. 2   The explanation was correct, demonstrated correct mathematical
               knowledge, and demonstrated a strong understanding of the material.
    1   The answer was correct but the explanation lacked appropriate mathematical
               terminology.
    0   The answer was incorrect

A PARTING WORD FOR THE CALCULUS TEACHER

This lesson has been designed for use with pre-calculus and calculus. The calculus teacher should feel free to place the emphasis on the derivative portion of the lesson. The emphasis should stress that the relative extrema occur where the derivative is undefined or equals zero. The addition of more functions representing those that have derivatives that are undefined at the extrema is probably a necessity for this purpose.